Bringing economic sense to economic capital in operational risk: the use of right truncated models for severity distribution.

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Outline

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Operational Risk Modeling

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Conclusions
Introduction

The technology that is available has increased substantially the potential of creating losses

Alan Greenspan, March 1995

Let us consider, at first, operational risk as risk arising from possible shortcomings in the routine operations of an entity (not only a bank).

Banking business has changed deeply.

The emergence of banks acting as large-volume service providers, deregulation, globalization, and advances in technology have increased complexity of bank activity and thus of their risk profile:

- Complex, multinational production processes,
- Complexity of financial products with numerous embedded options and guarantees.
- New business: for example banks are increasingly competing with insurers for asset products such as annuities and mutual funds or life insurance ...
- Large-scale mergers and acquisitions create risks from incompatible systems and integration problems.

New technologies create new risks:

- Automated back office processing systems increase risk of system failure;
- More automated hedging strategies (clearing and settlement systems) reduce market and credit risk but create additional operational risks;
- e-banking and e-commerce increase risk of fraud and create new and unknown risks;
- Outsourcing creates new risk exposures.
From Thick Fingers ...

- May 2001: an employee at Lehman Brothers negotiates an engagement of £300 millions instead of £3 millions (his real goal). This error implied a fall of 120 points of the FTSE 100 (≈ £40 billions).
- November 2001: another erroneous operation with EuroStoxx futures had as consequence a fall of 800 points of the index.
- December 2001: a trader at UBS Warburg made an error (in the Japanese equities book), typing the price (per unit) instead of the number of units resulting in a net loss of $50 millions.
- The automation of processes and the globalization of markets have converted these facts in usual.
- A trader may understand an order in an erroneous way and he is going to sell instead of buying. If in addition market moves the wrong way his error will result in a loss.
- In May 2002 there were more than 7,000 events of operational risk with losses of more than one million dollars each (a total of more than $272 billions).
… To Rogue Traders I

- **BCCI (1991, £27.000m: fraud):** The most amazing example of fraudulent use of a financial institution. Everything they did was wrong (criminal). Process began last year. There is a claim of £1 billion against the Bank of England.

- **Bankers Trust (1994, $150m: bad practice):** The bank was involved in a major trial by a user who accused it of improper business practices. The bank reached an extra judicial agreement with the other party but, however, suffered a serious reputational damage. It was later bought by Deutsche Bank.

- **Barings (1995, $1.300m + bankruptcy: unauthorized activity):** during 2 years, Nick Leeson (derivative trader) accumulated non reported losses.

- **In the case of Daiwa (1995, $1.100m) or Sumitomo (1996, $2.600m) the unauthorized activity was for a longer period of time: 11 and 3 years.**

- **Sumitomo (1996, $2.600m: unauthorized action):** Over more than three years, a copper trader registered accumulated losses. The bank’s reputation was seriously affected.
... To Rogue Traders II

- **Deutsche Morgan Grenfell (1996, $720m: unauthorized action):** a fund manager (Peter Young) did not respect its limits and was responsible for heavy losses which were offset by Deutsche Bank.

- **Natwest (1997, $127m: model error):** Kyriacos Papouis (a swaption trader) used wrong volatilities in the model for swaption pricing.

- **Cantor Fitzgerald and others (2001):** Attacks on the World Trade Center.

- **Merrill Lynch (2002 $100m: rogue trader).**

- **Allied Irish Bank (2003 $691m: unauthorized action):** A “rogue trader” at the U.S. subsidiary, hid three years of losses in the exchange yen/dollar desk. The bank’s reputation was seriously affected.

- **Bank of America, FleetBoston Financial (2004, $515m: bad practices):** penalty for “after-market trading”.

- **City Bank (2004, $2.7 billions):** extrajudicial agreement (**WorldCom** case).

- **City bank (2005, $2 billions):** extrajudicial agreement (**Enron** case).
... To Rogue Traders III

- **Société Générale**: €4500 millions. Rogue trader.
- **UBS (2011,$4.5 billions)**
- **Subprime Crisis.**

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**CDO de Alta Calificación**

- Senior AAA: 88%
- Junior AAA: 5%
- AA: 3%
- A: 2%
- BBB: 1%
- Sin Calificar: 1%

**CDO de Calificación Media**

- Senior AAA: 62%
- Junior AAA: 14%
- AA: 8%
- A: 6%
- BBB: 6%
- Sin Calificar: 4%

**CDO-squared**

- Senior AAA: 60%
- Junior AAA: 27%
- AA: 4%
- A: 3%
- BBB: 3%
- Sin Calificar: 2%

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*Fuente: The Institute for International Finance*

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**Aumento de complejidad**
How to Define Operational Risk?

- A direct reading of the quote above could lead to define it as any risk but market risk or credit risk.
- Indeed, reputational risk and systemic risk can not be considered as operational risk.
- At the other extreme, one can define it as the risk inherent in the transactions.
- These risks include: computer failures or systems, mistakes in negotiation.
- Also supervision errors (Barings), problems in the back office or in the models (one of the most notorious examples).
- In fact, most financial disasters meet a combination of exposure to market risk and/or credit with any failure of controls.
- One of the most serious problems is the difficulty of identifying operational risk: it doesn’t have such a well defined entry as have market or credit risk.
- At some moment in the past there were a debate about whether operational risk should be covered by Pilar 1 (economic capital) or 2 (supervision) eventually completed by insurance.
The Basel Committee Definition

- The above definition has the disadvantage focus on the issue of operations and to set aside such issues as:
  - Model risk,
  - Internal fraud,
  - Improper business practices or
  - Disasters such as the 11-S,
  - Damage to physical assets,
  - etc.

- The definition of operational risk by the Basel Committee (CP3, April 2003), is:

  ... the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This definition includes legal risk, but excludes strategic and reputational risk.
The Basel II Array

- The problem of computing economic capital for operational risk is very similar to insurance companies management.
- In order to allow a better computation, the Basel Committee proposed a classification based in 8 business lines:
  - Corporate Finance,
  - Trading & Sales,
  - Retail Banking,
  - Commercial Banking
  - Payment & Settlement,
  - Agency Services,
  - Asset Management,
  - Retail Brokerage.
- and 7 types of risk:
  - Internal Fraud,
  - External Fraud,
  - Employment Practices and Workplace Safety,
  - Clients, Products and Business Practices,
  - Damage to Physical Assets
  - Business Disruption and System Failures,
  - Execution, Delivery and Process Management
Severity and Frequency I

- The results of the LDCE 2002 (QIS 3) was remarkable: 47,269 losses of more than €20,000, representing gross losses of €8 billions.
- The distribution by business lines for the number of losses of QIS 3 is the following:

![Frequency by business line chart]

- Corporate finance: 12%
- Trading & sales: 7%
- Retail banking: 7%
- Commercial banking: 4%
- Payment & settlement: 3%
- Agency services: 2%
- Asset management: 1%
- Retail brokerage: 2%
Severity and Frequency II

- Looking at gross losses, we get a different figure:

![Severity by business line diagram]
Severity and Frequency III

- We can compare both situations in the following table:

<table>
<thead>
<tr>
<th>Business Line</th>
<th>Number of Events (%)</th>
<th>Gross Losses (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate finance</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Trading &amp; sales</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>Retail banking</td>
<td>64</td>
<td>30</td>
</tr>
<tr>
<td>Commercial Banking</td>
<td>7</td>
<td>29</td>
</tr>
<tr>
<td>Payment &amp; settlement</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Agency services</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Asset management</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Retail brokerage</td>
<td>7</td>
<td>12</td>
</tr>
</tbody>
</table>

- Let us consider now the different types of risk in the frequency dimension:
Severity and Frequency IV

Frequency by type of risk

- Internal Fraud: 35%
- External Fraud: 44%
- Employment Practices: 1%
- Clients, Products: 7%
- Damage to Physical Assets: 9%
- Business Disruption: 9%
- Execution, Delivery: 3%
Severity and Frequency V

- For the gross losses, the situation is quite different:
Severity and Frequency VI

- Information of both figure is grouped in this table:

<table>
<thead>
<tr>
<th></th>
<th>number of events (%)</th>
<th>gross losses (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal fraud</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>External Fraud</td>
<td>44</td>
<td>16</td>
</tr>
<tr>
<td>Employment</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>Clients, Products</td>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>Damage to</td>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>Business disruption</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Execution, delivery</td>
<td>35</td>
<td>29</td>
</tr>
</tbody>
</table>

- The situation is very different through the different cells (or units).
- For each cell, we must fit a distribution for severity and another one for frequency and then combine both in order to produce the corresponding aggregate loss distribution.
- Two kind of events: low frequency-high impact, high frequency-low impact.
About Loss Events

- The number of loss events in a big European bank is huge (~100,000 per year).
- Losses greater than €10,000 represent:
  - 50% of gross losses,
  - but less than 1% of the number of losses.
- Around 50% of the loss event represent losses smaller than €6-10.
- Operational losses appear to follow heavy-tailed tailed distributions.
- More than 90% of the capital charge is usually explained by a very small number of events.
- Data points span many orders of magnitude.
- The largest loss is usually at 30 (or more) standard deviations away from the mean.
- Many authors suggest the use of Extreme Value Theory distributions in order to fit real data.
Section

What is Operational Risk?

Operational Risk Modeling

Model Risk

Using Truncated Distributions

Conclusions
Introduction

- The distributions of operational risk losses present heavy tails.
- Often a small number of (high severity/low frequency) events have a large impact on capital.
- This observation has lead to the modeling of operational risk severity based on the use of:
  - Subexponential distributions, in particular the Pareto distribution.
  - Analytical formula based on an asymptotic approximation.
  - High thresholds in the data collection and/or modeling.
  - Extreme value theory approach (POT methodology or Pareto fitting of the tails).
- All of them can be helpful in a first approach to operational risk quantification.
- Nevertheless, they present some undesirable features which we investigate.
- We analyze, in a realistic framework setting of synthetical data, how the use of right-truncated distributions can avoid most of the drawbacks.
- Special attention is given to the dynamics of the risk measures, that is to their stability over time.
The Loss Distribution Approach

- We assume that the economic is capital calculated for some predetermined units/types-of-risk of the bank.
- Those units may be the different business lines or a different categorization specified by the bank.
- For each unit the severity and the frequency of losses are assumed to be independent, identically distributed random variables.
- Let N denote the number of events per unit of time (typically one year).
- And X the severity of losses (positive).
- Then the aggregate loss distribution is:

\[
S = \sum_{n=1}^{N} X_n \quad \text{where } X_1, \ldots, X_i, \ldots \text{ i.i.d, } \sim X
\]

- In this approach, a Capital-at-Risk figure is calculate, for each unit/type-of-risk, as the quantile 99.9%.
- The global (economic/regulatory) capital is the sum of those amounts.
- It is possible to take into account the dependence structure (diversification effects).
The Monte Carlo Scheme
Expected Loss Versus Unexpected Loss
Models for Frequency Distributions

- Let $N$ be the frequency of losses in one cell and $p_k = P(N = k)$.
- We say that $N$ belongs to the class $(a,b,0)$ if there are $a, b \in \mathbb{R}^+$ such that:
  \[
  \frac{p_k}{p_{k-1}} = a + \frac{b}{k}, \quad \forall k = 1, 2, \ldots
  \]
- Poisson distribution, binomial and negative binomial belong to this class:

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Expression</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson</td>
<td>$p_n = \frac{\lambda^n}{n!} e^{-\lambda}$</td>
<td>$\lambda &gt; 0$</td>
</tr>
<tr>
<td>Binomial</td>
<td>$p_n = \binom{N}{k} p^k (1-p)^{N-k}$</td>
<td>$N, k \in \mathbb{N}, p \in (0,1)$</td>
</tr>
<tr>
<td>Negative Binomial</td>
<td>$p_{k+r} = \binom{k+r-1}{r-1} p^r (1-p)^k$</td>
<td>$k \in \mathbb{N}, r &gt; 0, p \in (0,1)$</td>
</tr>
</tbody>
</table>

- Compound Poisson, binomial or negative binomial are other possibilities (not in class $(a,b,0)$).
Models for Severity Distributions

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Expression</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lognormal</td>
<td>$H(x) = N\left(\frac{\ln(x) - \mu}{\sigma}\right)$</td>
<td>$\mu, \sigma &gt; 0$</td>
</tr>
<tr>
<td>GEV</td>
<td>$H(x) = \exp\left(-\left[1 + \xi \frac{x - \alpha}{\beta}\right]_{+}^{-1/\xi}\right)$</td>
<td>$\alpha, \beta &gt; 0, \xi$</td>
</tr>
<tr>
<td>Pareto gen.</td>
<td>$H(x) = 1 - \left[1 + \xi \frac{x - \alpha}{\beta}\right]_{+}^{-1/\xi}$</td>
<td>$\alpha, \beta &gt; 0, \xi$</td>
</tr>
<tr>
<td>Weibull</td>
<td>$H(x) = 1 - \exp\left(-\left[\frac{x - \alpha}{\beta}\right]_{+}^{\xi}\right)$</td>
<td>$\alpha, \beta &gt; 0, \xi$</td>
</tr>
<tr>
<td>Gamma</td>
<td>$H'(x) = (x - \gamma)^{\alpha-1}\beta^\alpha \Gamma(\alpha)^{-1} e^{-(x-\gamma)/\beta}$</td>
<td>$\alpha &gt; 0, \beta &gt; 0, \gamma &gt; 0$,</td>
</tr>
<tr>
<td>Log-gamma</td>
<td>$X \sim \mathcal{LN}(\mu, \theta \times \sigma^2)$</td>
<td>$\theta \sim \Gamma(1/\beta, \beta)$</td>
</tr>
</tbody>
</table>

- Inverse gaussian, Burr, g-and-h distributions are other possibilities.
**g-and-h Distributions**

- This family of parametrical distributions has been introduced recently by Dutta and Perry.
- Consider $Z \sim \mathcal{N}(0,1)$ a standard normal random variable.
- A random variable $X$ is said to have a g-and-h distribution with parameters $a, b, g, h \in \mathbb{R}$, if $X$ satisfies
  \[
  X = a + b \frac{e^{gZ} - 1}{g} e^{hZ^2/2}
  \]
- A more general setting may be achieved by considering $g$ and $h$ to be polynomials including higher orders of $Z^2$.
- The parameters $g$ and $h$ govern the skewness and the heavy-tailedness of the distribution, respectively.
- In opinion of those authors, extreme value theory is not well adapted to operational risk framework while g-and-h distributions allow for a good description.
- Recent work by Degen et al. shows that for this kind of distributions, the convergence to Pareto distribution (asymptotical regime) is very slow.
The Real Dirty World

- In practice, we shall have more complicated situations.
- No single distribution fits well over the entire data set.

So we need mixtures of our distributions.
Subexponential Distributions

- Consider $X_1, \ldots, X_n, \ldots$, independent, identically distributed random variables with distribution function $F = F_X (X_i \sim X, \forall i)$.
- They belong to the class of subexponential distributions iff we have
  \[ \lim_{x \to \infty} \frac{P(X_1 + \cdots + X_n > x)}{P(\max(X_1, \ldots, X_n) > x)} = 1 \quad \text{for some (all) } n \geq 2 \]
- This means that severe overall losses are mainly due to a single large loss rather than the consequence of accumulated small independent losses.
- It can be shown that this equation is equivalent to:
  \[ \lim_{x \to \infty} \frac{F_{*n}(x)}{F(x)} = n \quad \text{for some (all) } n \geq 2 \quad (\bar{H}(x) = 1 - H(x)) \]
- A consequence is that, if the severity distribution $F$ is subexponential and
  \[ \sum_{n=0}^{\infty} (1 + \epsilon)^n P(N = n) < \infty, \text{ for some } \epsilon > 0 \]
  then $S$ is subexponential and
  \[ \bar{F}_S(x) \sim \mathbb{E}[N] \times \bar{F}(x), \quad x \to \infty \]
An Analytical Formula (Böcker-Klüppelberg)

- Let us suppose $F_S(x) \sim E[N]F(x)$.

- If $x_\kappa$ is such that $F_S(x_\kappa) = \kappa$, we shall have (single-loss approximation):
  \[
  \text{VaR}_\kappa(F_S) = x_\kappa \sim F^{-1}(1 - \frac{1 - \kappa}{E[N]}) \quad (1)
  \]

- Assuming we are in the asymptotic regime, we can compute the OpVaR.

- For example, for $E[N] = 100$, we get that $\text{VaR}_{99.999%}(F_S) \sim \text{VaR}_{99.999%}(F)$.

- The use of an additional mean correction term improve the result.

An example: the Pareto case

- Let $F_u$ be the peak over threshold (POT) distribution of the severity:
  \[
  F_u(x) = P(X-u \leq x/X > u) \quad \forall 0 \leq x < x_F-u, x_F = \sup\{x > 0/F(x) < 1\}
  \]

- An elementary calculation leads to $F(x) = F(u) \times F_u(x-u)$

- Using the Pareto approximation to the POT distribution (Balkema theorem), we get (for $\kappa \to 1$):
  \[
  F(x) \sim F(u) \times \left(1 + \frac{x - u}{\beta}\right)^{-1/\xi} \Rightarrow \text{VaR}_\kappa(G) \sim u + \frac{\beta}{\xi} \left[\frac{F(u)E[N]}{1 - \kappa}\right]^{\xi} - 1
  \]
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Conclusions
The Percentile 99.9

- One of the biggest issues in the LDA approach is the accurate calculation of the aggregate loss distribution 99.9 percentile.
- This requires a precise fit of the tail of the severity distribution.
- In practice, different distributions may offer similar goodness of fit to data with very different results in terms of capital:
  - different lognormals with high sigma,
  - g-and-h distributions,
  - Pareto distribution.
- The goal is to extrapolate the shape of the severity distribution far in the tail, based on the knowledge of part of the body.
- It may be very difficult (lack of sufficient data far in the tail) to distinguish between them.
- Model error is a real threat.
- Especially when high thresholds are used.
Severity Uncertainty I

- Data are sparse in the tails.
- There may not be enough empirical evidence to select model distributions with very different asymptotic behavior.
- Let us consider, for example:
  - a lognormal ($\mu = 10, \sigma = 2.5$), the histogram;
  - a piecewise defined distribution with a lognormal body and a g-and-h ($a = 0.5, b = 5 \times 10^4, g = 2.25$ and $h = 0.25$) tail (15% of data, $u_0 = 3 \times 10^5$);
  - a piecewise defined distribution with a lognormal body and a generalized Pareto ($u = u_0, \beta = 5 \times 10^5, \xi = 1$) tail (15% of data, $u_0 = 3 \times 10^5$).
- We are considering really heavy tailed distributions.
- In the following figures we compare the lognormal distribution (histogram) with the lognormal + h-and-g (left) and lognormal + Pareto (right).
Severity Uncertainty II

- The tail profiles of these distributions are very similar except very far in the tails.

- The asymptotic behaviors of the distributions are very different.
- Thus the CaR (or OpVar) associated to those distributions ($\lambda = 200$) are, respectively:
  - $1.42 \times 10^9$ (lognormal),
  - $6.21 \times 10^9$ (g-and-h) and
  - $1.54 \times 10^{10}$ (generalized Pareto),
Threshold Effect

Model Error

- To illustrate the effect of varying the left threshold on the error in the model, we generate 30,000 lognormal random numbers for different values of $\sigma$ ($\mu = 0$).
- Two threshold levels are chosen (6,000 and 12,000).
- We present the results of the best fit\(^1\) of severity distributions to data.

<table>
<thead>
<tr>
<th>Threshold</th>
<th>6,000</th>
<th>10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>Pareto</td>
<td>Pareto</td>
</tr>
<tr>
<td>1.00</td>
<td>Weibull</td>
<td>Weibull</td>
</tr>
<tr>
<td>1.25</td>
<td>Pareto</td>
<td>Pareto</td>
</tr>
<tr>
<td>1.50</td>
<td>Lognormal</td>
<td>GEV</td>
</tr>
<tr>
<td>1.75</td>
<td>Lognormal</td>
<td>Lognormal</td>
</tr>
<tr>
<td>2.00</td>
<td>Weibull</td>
<td>Lognormal</td>
</tr>
<tr>
<td>2.25</td>
<td>Lognormal</td>
<td>Pareto</td>
</tr>
<tr>
<td>2.50</td>
<td>Lognormal</td>
<td>Lognormal</td>
</tr>
</tbody>
</table>

- What’s about less well defined data.

\(^1\)using the distribution with the highest figure for the minimum of the p-values of the Kolmogorov-Smirnov and of the Anderson-Darling statistics.
Threshold Effect
Impact on Capital

- The impact on capital depends on the frequency of events.
- The frequency distribution must be corrected in order to take into account the probability mass of the losses under the left censoring threshold.
- For high frequencies, the impact on capital may be very important:

<table>
<thead>
<tr>
<th>Threshold</th>
<th>6,000</th>
<th>10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ</td>
<td>capital</td>
<td>variation</td>
</tr>
<tr>
<td>0.75</td>
<td>24,722,851</td>
<td>-45.14%</td>
</tr>
<tr>
<td>1.00</td>
<td>61,776,662</td>
<td>11.89%</td>
</tr>
<tr>
<td>1.25</td>
<td>64,931,049</td>
<td>-13.72%</td>
</tr>
<tr>
<td>1.50</td>
<td>114,193,654</td>
<td>6.92%</td>
</tr>
<tr>
<td>2.00</td>
<td>263,774,070</td>
<td>-11.75%</td>
</tr>
<tr>
<td>2.25</td>
<td>677,462,871</td>
<td>11.04%</td>
</tr>
<tr>
<td>2.50</td>
<td>1,825,327,187</td>
<td>14.31%</td>
</tr>
</tbody>
</table>
Critical Issues when Using Pareto Distribution (I)

\( \xi = 0.6 \)

- When fitting Pareto to actual loss data, it is usual to get high values for \( \xi \) (even greater than 1).
- The parameters estimates in the Pareto fit are unstable.
- The (absolute) fluctuations of economic capital are very important.
- In order to illustrate this, we generate (Pareto, \( \xi = 0.6 \)) 30 events greater than €10,000 quarterly and fit data to Pareto at the end of each period.
- It doesn’t seem to be an acceptable solution.
Critical Issues when Using Pareto Distribution (II)

$\xi = 1.1$

- For $\xi > 1$, the situation is even more dramatic.
- First, the expected value of the losses is infinite. Therefore one should expect problems of consistency in the calculation of economic capital.
- With real data it is very easy to get *extremely unrealistic amounts of capital*.
- In general, Pareto fits tend to overestimate the value of capital-at-risk.

![Graph showing relative variations of capital over quarters](image-url)
The Body Effect
Should Single-Loss Events Determine the Economic Capital?

- In a subexponential framework, high percentiles of the loss distributions levels are explained by a single extreme loss or a small amount of large losses.
- The value 99.9% is a very high percentile.
- Is it high enough in order to make the body (the part under the threshold) of the distribution irrelevant for the CaR calculation?
- In order to give an answer to this question, we perform the following simulation.
  - Random values of the loss severity are generated with a lognormal distribution ($\mu = 5, \sigma = 2$).
  - Different thresholds ($u$), determined by the probability of the tail ($p$), are chosen.
  - Three cases are considered:
    1. case 0: empirical data;
    2. case 1: the losses under the threshold $u$ are all equal to 0;
    3. case 2: the losses under the threshold $u$ are all equal to $u$;
- The results for the CaR are displayed in the following table.
- In the case of conditional CaR the tendencies are similar.
The Body Effect

<table>
<thead>
<tr>
<th></th>
<th>p</th>
<th>u</th>
<th>$\text{VaR}_0 \times 10^{-3}$</th>
<th>$\text{VaR}_1 \times 10^{-3}$</th>
<th>$\text{VaR}_2 \times 10^{-3}$</th>
<th>$\frac{\text{VaR}_2 - \text{VaR}_1}{\text{VaR}_0} \times 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 200$</td>
<td>0.50</td>
<td>149</td>
<td>1,253</td>
<td>1,249</td>
<td>1,263</td>
<td>1.14%</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>797</td>
<td>1,249</td>
<td>1,223</td>
<td>1,349</td>
<td>10.09%</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>1,946</td>
<td>1,256</td>
<td>1,204</td>
<td>1,557</td>
<td>28.05%</td>
</tr>
<tr>
<td>$\lambda = 2,000$</td>
<td>0.50</td>
<td>149</td>
<td>4,909</td>
<td>4,858</td>
<td>5,010</td>
<td>3.10%</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>797</td>
<td>4,896</td>
<td>4,624</td>
<td>5,897</td>
<td>25.98%</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>1,946</td>
<td>4,911</td>
<td>4,399</td>
<td>7,903</td>
<td>71.36%</td>
</tr>
<tr>
<td>$\lambda = 20,000$</td>
<td>0.50</td>
<td>149</td>
<td>28,655</td>
<td>28,126</td>
<td>29,653</td>
<td>5.32%</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>797</td>
<td>28,567</td>
<td>25,853</td>
<td>38,660</td>
<td>44.83%</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>1,946</td>
<td>28,727</td>
<td>23,519</td>
<td>58,630</td>
<td>122.22%</td>
</tr>
</tbody>
</table>

Conclusions:

- For low frequencies, the contribution of the body of the distribution is not decisive.
- Nevertheless the greater is the frequency the larger is the contribution of the body of the distribution.
- The asymptotic approximation works well for small frequencies.
- Note however that, in these experiments, the probability mass of the losses under the threshold is the same in all cases.
- If the probability mass of the body is extrapolated from the tail fit, these figures would show a much larger variation.
Section

What is Operational Risk?

Operational Risk Modeling

Model Risk

Using Truncated Distributions

Conclusions
Reasons for the Use of Truncated Distributions

- Economic and regulatory capital should have economic sense.
- The use of heavy tailed distributions in an LDA framework may lead to results with no economic interpretation:
  - infinite expected losses,
  - very unstable estimates of CaR values,
  - diverging estimates of conditional CaR values.
- The losses of a bank may not be arbitrarily large.
- It seems reasonable that the sequence:
  
  Basic Approach $\rightarrow$ Standard Approach $\rightarrow$ Advanced Models Approach

leads to a reduction in the requirements of capital.
- As we shall see, the use of truncated distributions allows to surpass this problems.
- It is interesting to outline that the right truncation level may be high.
- The determination of the this level is an open question we consider in a first approach here.
The Framework

- Since the asymptotic limit in which the CaR is dominated by a single extreme loss is reached earlier at lower the frequencies, we focus on a cell with only sporadic loss events.
- We suppose the frequency of losses is Poisson with an average of 200 (\( \lambda = 200 \)) losses per year.
- For the severity, we shall use the three distributions previously indicated:
  - a lognormal (\( \mu = 10, \sigma = 2.5 \));
  - a piecewise defined distribution with a lognormal body and a g-and-h (\( a = 0.5, b = 5 \times 10^4, g = 2.25 \) and \( h = 0.25 \) tail (15% of data, \( u_0 = 3 \times 10^5 \)));
  - a piecewise defined distribution with a lognormal body and a generalized Pareto (\( u = u_0, \beta = 5 \times 10^5, \xi = 1 \)) tail (15% of data, \( u_0 = 3 \times 10^5 \)).
- Several levels of right-truncation are considered.
The Lognormal Distribution

Truncated Case

Truncation level: $2 \times 10^8$

Truncation level: $2 \times 10^9$
The Lognormal Distribution

Non Truncated Case
The Lognormal + g-and-h Distribution
Truncated Case

Truncation level: $2 \times 10^8$

Truncation level: $2 \times 10^9$
The Lognormal Distribution + g-and-h Distribution

Non Truncated Case

![Graphs showing non-truncated cases of Lognormal Distribution and g-and-h Distribution](images)
The Lognormal + GP Distribution

Truncated Case

Truncation level: $2 \times 10^8$

Truncation level: $2 \times 10^9$
The Lognormal + GP Distribution

Non Truncated Case

![Graphs showing the Lognormal + GP Distribution for the non-truncated case.](image)
Summary

- These results are summarized in the table.

<table>
<thead>
<tr>
<th>Threshold</th>
<th>LN</th>
<th>LN + GH</th>
<th>LN + GP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \times 10^6$</td>
<td>$4.01 \times 10^7$</td>
<td>$4.03 \times 10^7$</td>
<td>$4.07 \times 10^7$</td>
</tr>
<tr>
<td>$4 \times 10^6$</td>
<td>$5.86 \times 10^7$</td>
<td>$5.90 \times 10^7$</td>
<td>$5.82 \times 10^7$</td>
</tr>
<tr>
<td>$1 \times 10^7$</td>
<td>$9.25 \times 10^7$</td>
<td>$9.45 \times 10^7$</td>
<td>$8.93 \times 10^7$</td>
</tr>
<tr>
<td>$2 \times 10^7$</td>
<td>$1.27 \times 10^8$</td>
<td>$1.35 \times 10^8$</td>
<td>$1.23 \times 10^8$</td>
</tr>
<tr>
<td>$4 \times 10^7$</td>
<td>$1.71 \times 10^8$</td>
<td>$1.88 \times 10^8$</td>
<td>$1.68 \times 10^8$</td>
</tr>
<tr>
<td>$1 \times 10^8$</td>
<td>$2.54 \times 10^8$</td>
<td>$2.92 \times 10^8$</td>
<td>$2.67 \times 10^8$</td>
</tr>
<tr>
<td>$2 \times 10^8$</td>
<td>$3.50 \times 10^8$</td>
<td>$4.22 \times 10^8$</td>
<td>$3.89 \times 10^8$</td>
</tr>
<tr>
<td>$4 \times 10^8$</td>
<td>$4.91 \times 10^8$</td>
<td>$6.11 \times 10^8$</td>
<td>$5.78 \times 10^8$</td>
</tr>
<tr>
<td>$1 \times 10^9$</td>
<td>$8.76 \times 10^8$</td>
<td>$1.08 \times 10^9$</td>
<td>$1.06 \times 10^9$</td>
</tr>
<tr>
<td>$2 \times 10^9$</td>
<td>$1.23 \times 10^9$</td>
<td>$1.84 \times 10^9$</td>
<td>$1.89 \times 10^9$</td>
</tr>
<tr>
<td>$\infty$</td>
<td>$1.42 \times 10^9$</td>
<td>$6.21 \times 10^9$</td>
<td>$1.54 \times 10^{10}$</td>
</tr>
</tbody>
</table>
Recapitulation

- The use of right-truncated distributions allows more stable estimations of economic/regulatory capital.
  - The differences between the various models are less pronounced.
  - For very heavy-tailed distributions the capital estimates are less sensitive to variations in the truncation level than to the sampling fluctuations when there is no truncation.

- The fact they imply less economical capital suggests that a careful analysis is needed in order to determine a reasonable truncation level.

- In the case of business lines/type of risk, a possibility is to take the capital required for the business line for the Standard Approach:
  - A bank having a greater loss in its data base will not be allowed for a smaller amount of capital.
  - However, this procedure by itself doesn't guarantee that the capital is smaller than using the Standard Approach.
  - Something not unusual.

- It is necessary to have a look at the dependence structure.
Looking at correlations

- In an LDA model, the dependence between aggregate losses is low: a 100% correlation between $N_1$ and $N_2$ implies less than 6% correlation between

$$S_{N_1} = \sum_{1}^{N_1} X_i \quad \text{and} \quad S_{N_2} = \sum_{1}^{N_2} Y_j$$

- Because of this, a diversified model build on frequencies implies a huge diminution of the regulatory capital.

- Most of regulators are more comfortable putting a (provisional) floor to the capital reduction.

- It makes a lot of sense look at the “correlation translation” of this restriction.

- What is the level of correlation between aggregate losses which would give the corresponding amount of capital?
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Conclusions
Conclusion

- Dutta and Perry introduce a qualitative yardstick against which any capital charge model ought to be tested:
  1. Good Fit - Statistically, how well does the method fit the data?
  2. Realistic - If a method fits well in a statistical sense, does it generate a loss distribution with a realistic capital estimate?
  3. Well Specified - Are the characteristics of the fitted data similar to the loss data and logically consistent?
  4. Flexible - How well is the method able to reasonably accommodate a wide variety of empirical loss data?
  5. Simple - Is the method easy to apply in practice, and is it easy to generate random numbers for the purposes of loss simulation?

- The methodology we propose satisfies all of these requirements.
- In addition it provides a framework that reduces the occurrence and mitigate consequences of model error.
- More research is necessary in order to identify a satisfactory level for truncation.
Muito Obrigado
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